On valuing corporate debt with the volatility of corporate assets evolving according to an Ornstein-Uhlenbeck process*

June 24, 2002

Bakhodir Ergashev
Department of Economics
Washington University in St. Louis
#1 Brooking Drive. 205 Eliot Hall.Campus Box 1208
St. Louis, MO 63130-4899
e-mail: ergashev@wueconc.wustl.edu

Abstract

In this paper the problem of valuing corporate debt with possibility of default is considered. It is assumed that the volatility of the value of a firm's assets evolves according to an Ornstein-Uhlenbeck process and default occurs only if the value of corporate assets falls below an exogenously specified, time dependent barrier. In the case of a particular choice of default barrier the explicit formulas for the present value of a corporate debt, the total value of the firm, the value of equity, the expected default time and the variation of default time are derived.

1 Introduction

Consider the following model of the value $V_b(t)$ of corporate assets:

$$\frac{dV_b(t)}{V_b(t)} = mdt + dY_b(t) \tag{1}$$

where

$$dY_b(t) = -bY_b(t)dt + \sigma dW_t, \qquad Y(b,0) = 0, \tag{2}$$

W_t is a Brownian motion and $m, \sigma > 0$, and $b \geq 0$ are constants.

*First draft. Comments, critique requested

If $b = 0$ process $Y_b(t)$ defined in (2) becomes a Wiener process and evolution of the value of the firm's assets is described by the following lognormal process:

$$\frac{dV_t}{V_t} = mdt + \sigma dW_t \tag{3}$$

where $V_t = V(0,t)$. This case has been extensively studied in many publications including Black and Cox(1976), Leland(1994), Goldstein, Ju, and Leland(2001), Longstaff and Schwartz(1995) and Briys and de Varenne(1997).
In the case of $b > 0$ process $Y_b(t)$ is a mean reverting Ornstein-Uhlenbeck process with zero mean. The case of small, positive values of b seems to be an interesting one for the following two reasons. First, in this case the volatility of the firm's value (although it can become arbitrarily large with appropriate choice of b) does not increase infinitely when $t \to \infty$. Second, small increments in the firm's value depend on its previous values through the autoregressive link defined in (2).
Throughout the paper we assume that the risk free interest rate r is a constant.
For simplicity we assume that the firm's debt is a consol bond with perpetual coupon payment, C, whose level remains constant unless the firm's value reaches or goes below default barrier. At that time the firm's debtholders can realize their right to bankrupt or force a reorganization of the firm.
Following Black and Cox(1976) we assume that default barrier is an exogenously specified, deterministic, time dependent function of an exponential form. We define default as the first time when the value of corporate assets hits default barrier $V_B exp(a(t))$:

$$\tau_a = inf\{t \geq 0 : \quad V_b(t) = V_B exp(a(t))\}$$

where $V_b(0) > V_B \geq 0; \quad a(0) = 0$, and V_B is a given constant.
Further, for simplicity of notations we use V instead of $V_b(0)$.
The two analytically tractable choices of default barrier are $a(t) \equiv 0$ studied by Leland(1994) under assumption (3) and $a(t) = r - \sigma^2/2$ studied in Section 2 below. Therefore our primary interest is in the following default times:

$$\tau = inf\{t \geq 0 : \quad V_t = V_B\}, \tag{4}$$

$$\tau_0 = inf\{t \geq 0 : \quad V_t = V_B exp\{(r - \sigma^2/2)t\} \quad \} \tag{5}$$

and

$$\tau_b = inf\{t \geq 0 : \quad V_b(t) = V_B exp\{(r - \sigma^2/2)t\} \quad \}, \quad b > 0, \tag{6}$$

In this paper we study corporate debt value under the framework (1)and (2) with default barrier defined by (5) (when $b = 0$) or (6) (when $b > 0$). Following Leland(1994) we assume that standard assumptions made in that paper are held. For the sake of clarity we study the cases $b = 0$ and $b > 0$ separately. We also demonstrate consistency of our results by showing that the results of the case $b = 0$ can be obtained from the case $b > 0$ by taking limit $b \to 0$.

In the case of $b > 0$ we are able to compute the discounted present value of an asset that pays one dollar at time of default τ_b due to the exponential family of martingales found by Novikov(1990). This key finding allows us to compute the other values of interest such as the value of the consol bonds, the value of tax benefits associated with debt financing, the total value of the firm etc. Some extensions of the results, such as the case of two-sided barrier[1] and the case of discrete time, are also discussed.

The structure of the paper is as follows. In Section 2 we study the case $b = 0$. First, we rederive Leland's(1994) results in our framework. Then we value corporate debt under the assumption that default barrier is defined by (5). In Section 3 we consider the case $b > 0$ and extend our results from previous section to this case with default barrier defined in (6). Section 4 concludes. In the Appendix we study the expected values and variations of the default times considered in this paper and present the explicit formulas for them whenever it is possible.

2 Pure diffusion volatility

2.1 The Leland's case

Assume that (3) holds and define risk neutral measure Q so that process $W_t^Q = \frac{m-r}{\sigma}t + W_t$ is a standard Wiener process (under Q). Then, using Ito's lemma, one can find the following presentation for V_t:

$$V_t = V exp\{(r - \sigma^2/2)t + \sigma W_t^Q\}.$$

Therefore τ, defined in (4), can be rewritten as

$$\tau = inf\{t \geq 0 : \quad (r - \sigma^2/2)t + \sigma W_t^Q = log(V_B/V)\}.$$

Also note that, due to the Novikov's condition, process

$$M_{1t} = exp\{-\frac{2r}{\sigma}W_t^Q - \frac{2r^2}{\sigma^2}t\} = exp\{-rt - \frac{2r}{\sigma^2}[(r - \sigma^2/2)t + \sigma W_t^Q]\}$$

is a Q-martingale. Now one can use the following identity[2] $E_0^Q[M_{1\tau}] = E_0^Q[M_{10}] \equiv 1$ to find

$$E_0^Q[exp(-r\tau)] = (V/V_B)^{-\frac{2r}{\sigma^2}}.$$

Here and further E_t^Q stands for the mathematical expectation under probability measure Q and information available at time t. Using this identity one can derive formulas (7), (9), (11)-(13) of Leland(1994). For example, at time $t = 0$ the value $D(V)$ of debt,

[1] In this case, when the value of the firm riches the upper bound, capital restructuring occurs. Optimal upward dynamic capital structure strategy with two sided barriers, when firm value evolves according to a lognormal distribution, is considered in Goldstein, Ju and Leland (2001).

[2] This and the other analogous identities presented further follow from the optional stopping theorem for martingales (see, for example, Liptser and Shiryaev(1989))

that promises a perpetual coupon payment C (unless the firm declares bankruptcy) and a fraction $1-\alpha$ of firm's value if bankruptcy occurs, equals

$$D(V) = E_0^Q[\int_0^\tau e^{-rt}Cdt + e^{-r\tau}(1-\alpha)V_B] = \frac{C}{r} + [(1-\alpha)V_B - \frac{C}{r}](V/V_B)^{-\frac{2r}{\sigma^2}}.$$

2.2 Time dependent default barrier

Now consider default time τ_0. Under Q-measure, defined above, τ_0 can be rewritten as

$$\tau_0 = inf\{t: \quad \sigma W_t^Q = log(V_B/V)\}$$

Using Q-martingale $M_{2t} = exp\{-\sqrt{2r}W_t^Q - rt\}$ and the identity $E_0^Q[M_{2\tau_0}] = E_0^Q[M_{20}] \equiv 1$ one can find that

$$E_0^Q[exp(-r\tau_0)] = (V/V_B)^{-\frac{\sqrt{2r}}{\sigma}}. \tag{7}$$

Therefore, when default time is given by τ_0, the price of a consol bond $B_0(V)$, with perpetual coupon payments C unless the firm is bankrupt, equals

$$B_0(V) = E_0^Q[\int_0^{\tau_0} e^{-rt}Cdt] = \frac{C}{r}[1 - (V/V_B)^{-\frac{\sqrt{2r}}{\sigma}}].$$

Assuming that in the case of bankruptcy a fraction $0 \leq \alpha \leq 1$ of value will be lost to bankruptcy costs we can express the value of bankruptcy cost $BC_0(V)$ as

$$BC_0(V) = E_0^Q[\alpha V_{\tau_0} exp\{-r\tau_0\}] = \alpha V_B E_t^Q[exp\{-\frac{\sigma^2}{2}\tau_0\}].$$

Since process $M_{3t} = exp\{-\sigma W_t^Q - \frac{\sigma^2}{2}t\}$ is a Q-martingale, one can find that

$$E_0^Q[exp\{-\frac{\sigma^2}{2}\tau_0\}] = V_B/V. \tag{8}$$

Therefore

$$BC_0(V) = \alpha V_B^2/V.$$

Due to the protective covenant the bondholders get the following value in the case of bankruptcy

$$E_0^Q[(1-\alpha)V_{\tau_0} exp\{-r\tau_0\}] = (1-\alpha)BC_0(V) = (1-\alpha)V_B^2/V.$$

Therefore the value of debt, that promises a perpetual coupon payment C (unless the firm declares bankruptcy) and a fraction $1-\alpha$ of firm's value at time of bankruptcy, can be expressed as

$$D_0(V) = E_t^Q[\int_0^{\tau_0} e^{-rt}Cdt + e^{-r\tau_0}(1-\alpha)V_B] = \frac{C}{r}[1 - (V/V_B)^{-\frac{\sqrt{2r}}{\sigma}}] + (1-\alpha)V_B^2/V.$$

Following Leland(1994) assume that tax benefits associated with debt financing pay a constant proportion of interest payments (δC, where $0 \leq \delta \leq 1$) as long as the firm is solvent and pay nothing in bankruptcy. Then the value of this security $TB_0(V)$ equals

$$TB_0(V) = E_0^Q[\int_0^{\tau_0} \delta C exp\{-rt\}dt] = \delta B_0(V) = \frac{\delta C}{r}[1-(V/V_B)^{-\frac{\sqrt{2r}}{\sigma}}].$$

Now, the total value of the firm, $v_0(V)$, defined as the firm's asset value (V) plus the value of tax deductions of coupon payments ($TB_0(V)$) less the value of bankruptcy costs ($BC_0(V)$), can be expressed as

$$v_0(V) = V + \frac{\delta C}{r}[1-(V/V_B)^{-\frac{\sqrt{2r}}{\sigma}}] - \alpha \frac{V_B^2}{V}.$$

The value of equity, $E_0(V)$, then equals

$$E_0(V) = v_0(V) - D_0(V) = V - (1-\delta)\frac{C}{r}[1-(V/V_B)^{-\frac{\sqrt{2r}}{\sigma}}] - \alpha \frac{V_B^2}{V}.$$

3 Ornstein-Uhlenbeck volatility

Further assume that $b > 0$ and the firm's assets value $V_b(t)$ evolves according to (1) and (2). The debtholders receive perpetual continuous coupon payments at rate C as long as the firm remains solvent and default time τ_b is defined by (6). In Section 3.3 under the above given assumptions we present the explicit formulas for the values of corporate debt, equity and the firm's total value. But, before that, as in the previous section, first we need to find an appropriate risk neural probability measure $Q(b)$ under which all assets have the same risk free payoff r (Section 3.2). Second, we need to find the appropriate $Q(b)$- martingales that enable us to price an asset that pays \$1 at time of bankruptcy ($E_0^{Q(b)}[e^{-r\tau_b}]$) and a claim for the firm's assets value at time of bankruptcy ($E_0^{Q(b)}[e^{-r\tau_b}V(b,\tau_b)]$). This is done in Section 3.2. In Section 3.4 we demonstrate the consistency of our results by showing that findings of Section 2.2 is the limit case of the findings of Section 3.3 as $b \to 0$.

3.1 Change of measure

Using Ito' lemma one can find the following solution to (1):

$$V_b(t) = V exp\{(m - \sigma^2/2)t + Y_b(t)\} \tag{9}$$

Now we define the risk neutral probability measure $Q(b)$. Let $Q(b)$ be chosen so that the process $W_t^{Q(b)}$ defined by

$$dW_t^{Q(b)} = dW_t + \frac{m-r}{\sigma}(bt+1)dt, \tag{10}$$

is a $Q(b)$- Brownian motion.
Note that when $b = 0$ measure $Q(b)$ becomes Q measure defined in Section 2.
Define the following process

$$Z_t = \sigma \int_0^t exp\{-b(t-s)\} dW_t^{Q(b)}, \qquad \text{with} \quad Z_0 = 0. \tag{11}$$

Due to its definition and Ito's lemma Z_t satisfies the following stochastic differential equation

$$dZ_t = -bZ_t dt + \sigma dW_t^{Q(b)}.$$

From (10) and (11) it is easy to see that

$$Z_t = (m-r)t + Y_b(t). \tag{12}$$

From (9) and (12) we find

$$V_b(t) = V exp\{(r - \frac{1}{2}\sigma^2)t + Z_t\}. \tag{13}$$

The last presentation shows that under $Q(b)$ measure $V_b(t)$ is a solution to the following stochastic differential equation

$$\frac{dV_b(t)}{V_b(t)} = rdt + dZ_t.$$

Earlier Ergashev(2002a) used probability measure defined in (10) to find the price of an European call option with the value of underlying security evolving according to (1).

3.2 An exponential family of martingales

The following Corollary immediately follows from Proposition 3 of Novikov(1990).

Corollary 1.
The processes

$$N_t(\mu) = exp\{-b\mu t\} \int_0^\infty exp\{-uZ_t - \frac{\sigma^2}{4b}u^2\} u^{\mu-1} du, \quad \mu > 0 \tag{14}$$

are the $Q(b)$-martingales.

Note 1. One can check the martingale property of $N_t(\mu)$ directly using the formula

$$E_s^{Q(b)}[exp\{-uZ_t\}] = exp\{-uZ_s e^{b(s-t)} + \frac{\sigma^2 u^2}{4b}(1 - e^{2b(s-t)})\}$$

that is based on the following integral presentation of Z_t:

$$Z_t = Z_s e^{b(s-t)} + \sigma \int_s^t e^{b(v-t)} dW_v^{Q(b)}, \qquad s \le t.$$

The properties of the exponential family of martingales, that includes $N_t(\mu)$ defined in (14) as a particular case, are studied in Novikov(1989). Some applications of this family of martingales are studied in Novikov(1990) and Novikov and Ergashev(1994).

3.3 Corporate debt and equity values

Using (13) we can rewrite τ_b as

$$\tau_b = inf\{t \geq 0 : \quad Z_t = ln(V_B/V)\}.$$

For simplicity of further notations we denote $G(u,\mu) = u^{\mu-1}exp\{-\frac{\sigma^2}{4b}u^2\}$.
Now we formulate and prove our main result.

Proposition 1. *Under assumptions made in the Introduction the followings are true*

$$E_0^{Q(b)}[e^{-r\tau_b}] = \frac{\int_0^\infty G(u,r/b)du}{\int_0^\infty (V/V_B)^u G(u,r/b)du}, \tag{15}$$

and

$$E_0^{Q(b)}[e^{-r\tau_b}V_b(\tau_b)] = V_B \frac{\int_0^\infty G(u,\nu)du}{\int_0^\infty (V/V_B)^u G(u,\nu)du}, \tag{16}$$

where $\nu = \sigma^2/(2b)$.

Proof.
From the Corollary and the optional stopping theorem for martingales we have:
$E_0^{Q(b)}[N_{\tau_b}(\mu)] = E_0^{Q(b)}[N_0(\mu)]$.
Due to the definition of τ_b this identity can be rewritten as

$$E_0^{Q(b)}[e^{-b\mu\tau_b}] = \frac{\int_0^\infty G(u,\mu)du}{\int_0^\infty (V/V_B)^u G(u,\mu)du}. \tag{17}$$

Now (15) follows from (17) with $\mu = r/b$.

To prove (16) we note that

$$V_b(\tau_b) = V_B exp\{(r - \sigma^2/2)\tau_b\}. \tag{18}$$

Therefore

$$E_0^{Q(b)}[e^{-r\tau_b}V_b(\tau_b)] = V_B E_0^Q[e^{-\frac{\sigma^2}{2}\tau_b}]. \tag{19}$$

Now (16) follows from (19) and (17) with $\mu = \nu$. Q.E.D.

Due to Proposition 1 we are able to derive the following explicit formulas for bankruptcy cost $(BC_b(V))$, corporate debt value $(D_b(V))$, the value of tax benefits $(TB_b(V))$, the total value of the firm ($v_b(V)$) and the value of equity ($E_b(V)$):

$$BC_b(V) = \alpha\Omega(\nu),$$

$$D_b(V) = \tfrac{C}{r}[1 - \Omega(r/b)] + (1-\alpha)V_B\Omega(\nu),$$

$$TB_b(V) = \tfrac{\delta C}{r}[1 - \Omega(r/b)],$$

$$v_b(V) = V + \tfrac{\delta C}{r}[1 - \Omega(r/b)] - \alpha V_B\Omega(\nu),$$

$$E_b(V) = V - (1-\delta)\tfrac{C}{r}[1 - \Omega(r/b)] - \alpha V_B\Omega(\nu),$$

where $\Omega(\mu) = \int_0^\infty G(u,\mu)du / \int_0^\infty (V/V_B)^u G(u,\mu)du$.

3.4 The link between cases $b = 0$ and $b > 0$

Since by definition $lim_{b\to 0}\tau_b = \tau_0$ and $lim_{b\to 0}V_b(t) = V_t$ we should expect the formulas of Section 3.3 to be consistent with those of Section 2.2.
Since all findings of Section 2.2 are based on (7) and (8), to show this it is sufficient to prove that

$$lim_{b\to 0}E_0^{Q(b)}[exp(-r\tau_b)] = (V/V_B)^{-\frac{\sqrt{2r}}{\sigma}}.$$

and

$$lim_{b\to 0}E_0^{Q(b)}[exp(-r\tau_b)V_b(\tau_b)] = V_B/V.$$

For that purpose we use the Laplace's formula described below (for more details about the Laplace's method see, for example, Chapter 2 of Wong(1989)).
Suppose ϕ and h are continuous functions and h is also twice differentiable in interval (c,d). Assume that h has the maximum point ξ in (c,d) such that $h'(\xi) = 0$ and $h''(\xi) < 0$. Then according to the Laplace's formula

$$\int_c^d \phi(u)e^{Ah(u)}du \approx \sqrt{\frac{-2\pi}{Ah''(\xi)}}\phi(\xi)e^{Ah(\xi)} \quad \text{as} \quad A\to\infty. \tag{20}$$

We take $c = 0$; $d = \infty$; $A = 1/b$; $\phi_1(u) = 1/u$; $\phi_2(u) = (V/V_B)^u/u$ and $h(u) = rlog(u) - \frac{\sigma^2}{4}u^2$.

Then obviously

$$\int_0^\infty G(u,\mu)du = \int_0^\infty \phi_1(u)e^{Ah(u)}du, \tag{21}$$

and

$$\int_0^\infty (V/V_B)^u G(u,\mu)du = \int_0^\infty \phi_2(u)e^{Ah(u)}du. \tag{22}$$

Since in our case $\xi = argmax_{0 \le u < \infty} h(u) = \frac{\sqrt{2r}}{\sigma}$, $\phi_1(\xi) = \sigma/\sqrt{2r}$, and $\phi_2(\xi) = (\sigma/\sqrt{2r})(V/V_B)^{\frac{\sqrt{2r}}{\sigma}}$, using Laplace's formula (20) we can find from (21) and (22) that

$$lim_{b \to 0} \Omega(r/b) = \phi_1(\xi)/\phi_2(\xi) = (V/V_B)^{-\frac{\sqrt{2r}}{\sigma}}.$$

Analogously one can show that $lim_{b \to 0} \Omega(\nu) = V_B/V$.

4 Conclusion and Discussions

Under the assumptions described in the Introduction we have derived the exact formulas for corporate debt value, the value of equity and the total value of a corporate firm when default barrier has the form $V_B exp\{(r - \sigma^2/2)t\}$.
The results can easily be extended to the case of disctere time. It should be noted, however, that in that case firm's value at default time not exactly equals the level of default barrier. The difference between them begins playing a significant role. Ergashev(2002) discusses the ways of dealing with this problem in a different setting.
Following Goldstein, Ju, and Leland(2001) one can consider a two sided default barrier. This allows one to capture the possibility of capital structuring when firm's value reaches the upper barrier. However, it seems that when $b > 0$ the closed form solutions can be obtained in the case of upper and lower barriers of the form $V_i exp\{(r - \sigma^2/2)t\}$, $i = 1, 2$ only.

5 Appendix: The default times' expected values and variations

Here we study the expected value and variation of the default times presented in the Introduction. These two variables are important since they can partially characterize the distribution of default time.
If $r < \sigma^2/2$ then one can use Q - martingales W_t^Q and $(W_t^Q)^2 - t$ to find the following formulas for the expected value and variation of default time τ :

$$E_0^Q(\tau) = \frac{log(V/V_B)}{\sigma^2/2 - r}, \qquad Var_0^Q(\tau) = \sigma^2 \frac{log(V/V_B)}{(\sigma^2/2 - r)^3}.$$

If $r \ge \sigma^2/2$ then obviously both the expected value and variation of τ do not exist.
Default time τ_0 also does not have finite expected value and variation. In contrast, at it is shown below, both the expected value and variation of τ_b $(b > 0)$ are finite.
But, one can show (using asymptotic expansions of the below given formulas) that $E_0^{Q(b)}(\tau_b)$ and $Var_0^{Q(b)}(\tau_b)$ become infinitely large as $b \to 0$.

Now we present the explicit formulas for the expected value and variation of default time τ_b, when $b > 0$. These formulas are given in Proposition 2.
First, the appropriate martingales are presented in Corollary 2. Then the explicit formulas of Proposition 2 are derived from the optional stopping theorem applied to those martingales.

Corollary 2.
The processes

$$H_t = \int_0^\infty u^{-1}(exp\{-uZ_t\}-1)exp\{-\frac{\sigma^2}{4b}u^2\}du-bt \tag{A.1}$$

and

$$F_t = \int_0^\infty u^{-1}(exp\{-uZ_t\} - 1)exp\{-\frac{\sigma^2}{4b}u^2\}log(u)du$$

$$-bt\int_0^\infty u^{-1}(exp\{-uZ_t\}-1)exp\{-\frac{\sigma^2}{4b}u^2\}du-C_1t+\frac{b^2}{2}t(t+1), \tag{A.2}$$

where

$$C_1 = \int_0^\infty u^{-1}[exp\{-\frac{\sigma^2}{4b}u^2e^{-2b}\}-exp\{-\frac{\sigma^2}{4b}u^2\}]log(u)du,$$

are the $Q(b)$-martingales.

Note 2: The prove of the martingale property of H_t is given in Proposition 4 of Novikov(1990). One can check the martingale property of H_t directly, as explained in Note 1 of Section 3. However, here one also needs the identity

$$\int_0^\infty u^{-1}[exp\{-\frac{\sigma^2u^2}{4b}e^{2b(s-t)}\} - exp\{-\frac{\sigma^2u^2}{4b}\}]du = -b(s-t).$$

This identity is a particular case of the well-known Frullani identity.
The martingale F_t is a particular case of martingale given by formula (1.10) in Novikov and Ergashev(1994). The precise prove of the martingale property of F_t is given in Ergashev(1991) and available from the author upon request.

Proposition 2
When default time is defined by τ_b the expected default time equals

$$E_0^{Q(b)}(\tau_b) = \frac{1}{b}\int_0^\infty u^{-1}[(V/V_B)^u - 1]exp\{-\frac{\sigma^2}{4b}u^2\}du, \tag{A.3}$$

and the variation of the expected default time equals

$$Var_0^{Q(b)}(\tau_b) = [E_0^{Q(b)}(\tau_b)]^2-[1-2C_1/b^2]E_0^{Q(b)}(\tau_b)-2C_2/b^2, \tag{A.4}$$

where

$$C_2 = \int_0^\infty u^{-1}[(V/V_B)^u - 1]exp\{-\frac{\sigma^2}{4b}u^2\}log(u)du.$$

Formulas (A.3) and (A.4) follow (after some algebraic rearrangements) from the optional stopping theorem for martingales applied to (A.1) and (A.2), correspondingly.

References

Black, F., and Cox, J.C. 1976. Valuing corporate securities: some effects of bond indenture provisions. *Journal of Finance* 31, 351-367.

Briys, E., and de Varenne, F. 1997. Valuing risky fixed rate debt: an extension. *Journal of financial and quantitative analysis* 32. 239-248.

Ergashev, B.A. 2002a. A note on a generalized Black-Scholes formula. Working Paper, Department of Economics, Washington University in St. Louis. Available at http://econwpa.wustl.edu/eprints/fin/papers/0203/0203006.abs

Ergashev, B.A. 2002b. A necessary condition for optimality of the EWMA control schemes and their average run length approximations. Submitted to *IIE Transactions: Quality and Reliability Engineering.* 22 p.

Ergashev, B.A. 1991. Some problems of the Sequential Statistical Analysis connected with detecting a change point. Unpublished Candidate of Science Dissertation. Steklov Institute of Mathematics of the Russian Academy of Sciences, Moscow, Russia. (Russian)

Goldstein, R., Ju, N., and Leland, H. 2001. An EBIT-based model of dynamic capital structure. *The Journal Of Business* 74, 483-512.

Leland, H. 1994. Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance* 49, 1213-1252.

Liptser, R.Sh., and Shiryaev A.N. 1989. Theory of martingales. Boston: Kluwer Academic Publishers. **Longstaff, F.A., and Schwartz, E.D.** 1995. A simple approach to value risky fixed and floating rate debt.*Journal of Finance* 50, 789-79?

Novikov, A. A. 1989. On a family of martingales related to autoregression process, in *Statistics and control of random processes (A.N. Shiryaev, ed.)*, "Nauka", Moscow. 158-171. (Russian)

Novikov, A. A. 1990. On the first passage time of an autoregressive process over a level and an application to a "disorder" problem. *Theory probability and its applications* 35. 269-279.

Novikov, A. A. and Ergashev, B. A. 1994. Limit theorems for the first passage time of an autoregressive process over a level, in *Statistics and control of random processes (A.A.Novikov and A.N. Shiryaev, ed.)*, Providence, R.I.: American Mathematical Society, USA. 169-186.

Wong, R. 1989. Asymptotic Approximations of integrals. Boston: Academic Press.